Analysis and Design of Structure

RCC BEAMS

RCC Beams

A beam is reinforced by placing steel bars in the tensile zone of the concrete beam so that the compressive bending stress is carried by concrete and tensile bending stress is carried entirely by steel reinforcing bars.

MODULAR RATIO

Consider composite section consisting of two bars, one of concrete and other of steel, well bonded together with the help of end plugs, and subjected to a compressive load P.

Let E_s = Modulus of elasticity of steel

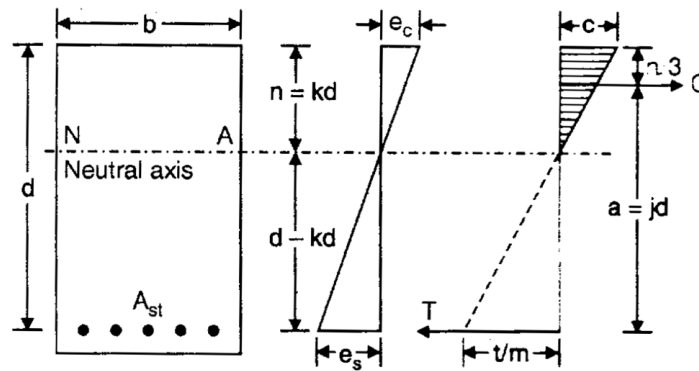 E_c = Modulus of elasticity of concrete.

Modular ratio will be

$$m = \frac{E_s}{E_c}$$

It means that load carried by steel is **m limes** the load carried by concrete.

Singly Reinforced Beam

NEUTRAL AXIS OF BEAM SECTION

(a) Section (b) Strain diagram (c) Stress diagram

Case 1: Stresses in Concrete and steel known

Let   c = compressive stress in the extreme fibre of concrete

        t = tensile stress in steel reinforcement b = breadth of beam

        d = depth to the centre of reinforcement (known as the effective depth)

        n = kd = depth of N.A. below the top of the beam

        k = neutral axis depth factor = n/d

        A_{st} = area of tensile reinforcement.

Since there is no resultant force across the section,

Total compression = total tension

Neglecting the tensile stress in concrete, we get

$$\frac{1}{2}c.b.kd = t.A_{st}$$

$$\frac{A_{st}}{b.d} = k.\frac{c}{2t}$$

$\therefore$ $\qquad k = \dfrac{mc}{mc+t}$ $\qquad$ (k is neutral axis depth factor)

Also, percentage steel p will be

$$p = \frac{A_{st}}{b.d}x100$$

or it can also be written as

$$p = \frac{50mc^2}{t(mc+t)}$$

Case 2: Dimensions of the beam completely known

Let us now take the case when the dimensions of the section, including the reinforcement are known, but the stresses in concrete and steel are not known.

Equating the moment of area in compression to the moment of the equivalent area in tension about the N.A., we get,

$$b.kd.\frac{kd}{2} = mA_{st}(d-kd)$$

MOMENT OF RESISTANCE

The total compressive force acting at the centre of gravity of the compressive forces is equal to the total tensile force acting at the centre of gravity of the steel reinforcement. Hence the moment of resistance of the reinforced concrete beam section is equal to the moment of the couple consisting of the compressive force and the tensile force.

The total compressive force

$$C = \frac{1}{2}c.k.b.d. = \frac{1}{2}\sigma_{cbc}.kd.b$$

acts at a distance of kd/3 from the top of the section.

Similarly, the total tensile force

$$T = t.A_{st} = \sigma_{st}.A_{st}$$ and acts at a distance d from the top of the section.

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A FOCUSSED APPROACH

Lever Arm

If jd is the lever arm (i.e. the distance between the compressive force and the tensile force), we have

$$a = jd = d - \frac{kd}{3} = d\left(1 - \frac{k}{3}\right)$$

Now, moment of resistance M_r is given by

$$M_r = \text{force x lever arm} = \frac{1}{2}c.kd.b.(jd) = \left(\frac{1}{2}cjk\right)bd^2 = Rbd^2$$

where $\qquad R = \frac{1}{2}cjk$

Also $\qquad M_r = t.A_{st}.jd = \sigma_{st}A_{st}.jd$

$\therefore \qquad A_{st} = \dfrac{M_r}{t.jd} = \dfrac{M}{\sigma_{st}.jd}$

## VALUES OF DESIGN CONSTANTS

Refer following table.

Concrete

Grade of concrete	M15	M20	M25	M30
Modular ratio (m)	18.67	13.33	10.98	9.33
σ_{cbc} (N/mm^2)	5	7	8.5	10

Steel

Grade of steel	Fe 250	Fe 415
σ_{cbc} (N/mm^2)	140	230

BALANCED SECTION

In a beam section, if the area of steel reinforcement A_{st} is of such magnitude that the permissible stresses c (σ_{cbc}) and t (σ_{st}) in concrete and steel respectively, are developed *simultaneously*, the section is known as the **balanced section**, **critical section or economical section.** For such a balanced section, the moment of resistance obtained from the compressive force will be equal to the moment of resistance obtained from the tensile force.
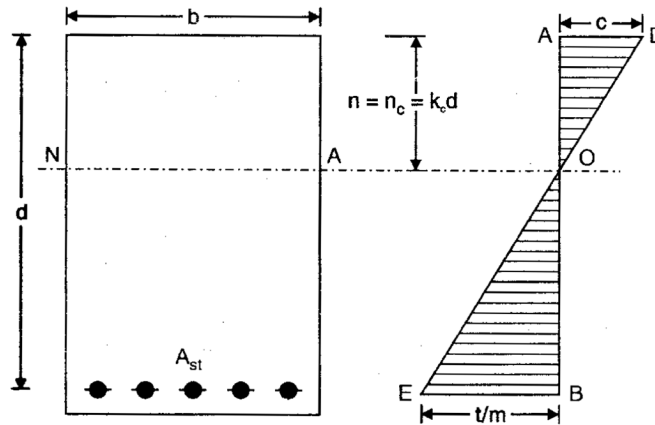
Percent area of reinforcement in balanced section

$$p = \frac{1}{2}k.\frac{c}{t}x100 = \frac{1}{2}.k.\frac{\sigma_{cbc}}{\sigma_{st}}x100$$

UNDER REINFORCED SECTION

An under reinforced section is the one in which the percentage steel provided is less than that given as $p = \frac{A_{st}}{b.d}x100$ and therefore full strength of concrete in compression is not developed.

The actual neutral axis of such a section will fall above the critical neutral axis of a balanced section.
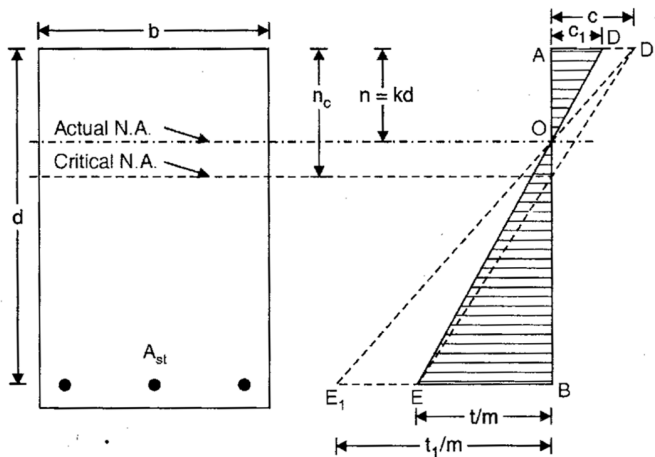
Given figure shows the under-reinforced section in which the actual N.A. is above the critical N.A.

In a under-reinforced concrete, the concrete is not fully stressed to its permissible value when stress in steel reaches its maximum value $t = \sigma_{st}$.

The moment of resistance of an under-reinforced section is, therefore, computed on the basis of the tensile force in steel:

$$M_r = tA_{st}.jd = \sigma_{st}.A_{st}.jd$$

OVER REINFORCED SECTION

In an over-reinforced section, the reinforcement provided is more than critical one and therefore the actual N.A. of such a section falls below the critical N.A. of a balanced section.
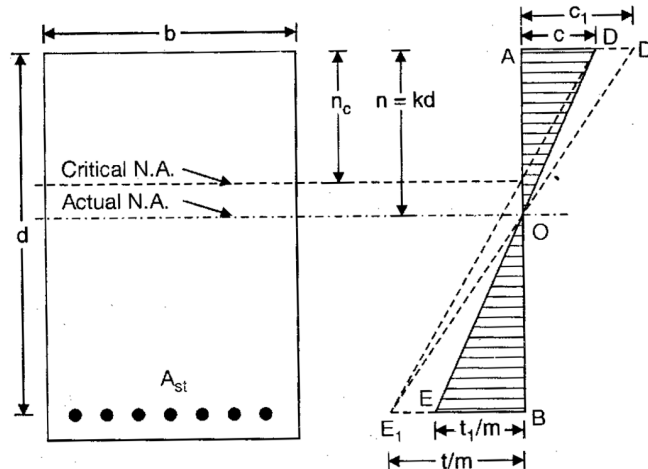
Thus, in a over-reinforced section, steel reinforcement is not fully stressed to its permissible value and the moment of resistance is determined on the basis of compressive force developed in concrete:

$$M_r = \frac{1}{2}c.kd.b.jd = \frac{1}{2}c.j.k.bd^2 = \frac{1}{2}\sigma_{cbc}.j.k.bd^2$$

Or it may also be written as

$$M_r = \frac{1}{2}c.n.\left(d - \frac{n}{3}\right)b = \frac{1}{2}\sigma_{cbc}.n\left(d - \frac{n}{3}\right)b$$

where n is depth of neutral axis.

TYPES OF PROBLEMS IN SINGLY-REINFORCED BEAMS

There may be three types of problems in singly reinforced concrete beams :

1. Determination of the moment of resistance of the given section

2. Determination of stresses in the given section subjected to a given bending moment.

3. Design of section to resist a given bending moment.

Example

For a balanced rectangular section (b x d) of a singly reinforced beam, determine (i) depth of neutral axis (ii) moment of resistance and (iii) percentage of steel using M 15 concrete and using Fe 415 steel. If b = 200 mm and d = 300 mm, determine the numerical values of n, M, and A_{st}.

Solution

Given data

$$c = \sigma_{cbc} = 5 \text{ N/mm}^2; \; m = 19; \; t = \sigma_{st} = 230 \text{ N/mm}^2$$

Depth of neutral section

For balanced section

$$k = \frac{mc}{mc+t} = \frac{19x5}{19x5+230} = 0.292$$

$\therefore$ Depth of neutral axis = n = kd = 0.292 x d

Moment of resistance

$$M_r = \frac{1}{2}cjk.bd^2 = \frac{1}{2}x5x0.903x0.292 = 0.659bd^2 \text{ N-mm}$$

Here, we calculated j as

$$j = \left(1 - \frac{k}{3}\right) = 1 - \frac{0.292}{3} = 0.903$$

Percentage of steel

$$p = \frac{50mc^2}{t(mc+t)} = \frac{50x19(5)^2}{230(19x5+230)} = 0.318\%$$

Numerical values of n, M, and A_{st}

$$n = 0.292 \text{ x } 300 = 87.6 \text{ mm}$$

$$M_r = 0.659(200)(300)^2 = 11.86 \text{ x } 10^6 \text{ N-mm} = 11.86 \text{ kN-m}$$

$$A_{st} = p.\frac{bd}{100} = \frac{0.318 x 200 x 300}{100} = 190.8 \, mm^2$$

Example

Determine the moment of resistance of a singly reinforced beam 160 mm wide and 300 mm deep to the centre of reinforcement, if the stresses in steel and concrete are not to exceed 140 N/mm² and 5 N/mm². The reinforcement consists of 4 bars of 16 mm diameter. Take m = 18. If the above beam is used over an effective span of 5 m, find the maximum load the beam can carry, inclusive of its own weight.

Given data

A_{st} $(\pi/4)$ x 16^2 = 804 mm^2; c = σ_{cbc} = 5 N/mm^2; t = σ_{st} = 140 N/mm^2; m = 18

Neutral axis

Equating the moment of inertia in compression to the moment of the equivalent area in tension about neutral axis (NA), we get

$$b \, x \, n \, x \frac{n}{2} = mA_{st}(d-n)$$

$\therefore$
$$\frac{160 \, x \, n^2}{2} = 18 \, x \, 804(300-n)$$

Solving this quadratic equation
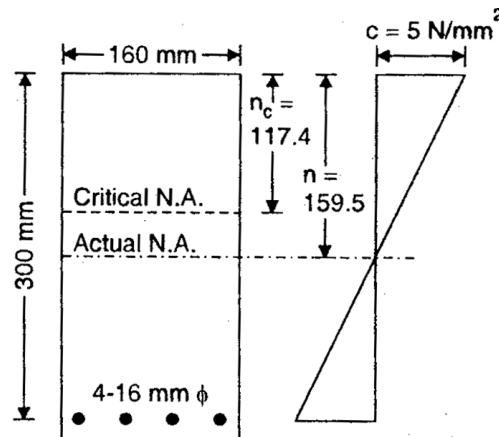
n = 159.5 mm

Depth of critical neutral axis

If n_c is the depth of critical neutral axis, we have

$$n_c = k.d = \frac{mc}{mc+t}.d$$

$\therefore$
$$n_c = \frac{18 \, x \, 5}{(18 \, x \, 5)+140} \, x \, 300 = 117.4 \, mm$$

Stress diagram

Moment of resistance

Since the depth of actual neutral axis is more than the critical one, the section is **over reinforced**. Thus, concrete reaches its maximum stress earlier to steel. Hence the moment of resistance is found on the basis of compressive force developed in concrete.

$$\text{Lever arm} = d - \frac{n}{3} = 300 - \frac{159.5}{3} = 246.8 \, mm$$

$$\therefore \qquad M_r = \frac{1}{2} c.n.b \left(d - \frac{n}{3} \right) = \frac{1}{2} x\,5\,x\,159.5\,x\,160(246.8) = 15.75 \, kN - m$$

Maximum bending moment and maximum load

It will be

$$= \frac{wl^2}{8} = \frac{wx25}{8} kN - m$$

Equating this to the moment of resistance of the beam, we get

$$w = \frac{15.75x8}{25} = 5.04 \, kN / m$$

Problem

A singly reinforced concrete beam with an effective span of 4 m has a rectangular cross sections having a width of 250 mm and an overall depth of 450 mm. The beam is reinforced with 3 bars of 10 mm diameter Fe 415 HYSD bars at an effective depth of 400 mm. The self weight of the beam together with dead load on the beam is 4 kN/m. Calculate the maximum permissible live load on the beam assuming M15 grade concrete.

Answer: 5.96 kN/m

Hint: σ_{st} = 230 N/mm²

Example

The cross-section of a simply supported reinforced beam is 200 mm wide and 300 mm deep to the centre of the reinforcement which consists of 3 bars of 16 mm dia. Determine from the first principles the depth of NA and the maximum stress in concrete when steel is stressed to 120 N/mm². Take m = 19.

Solution

Given data

b = 200 mm; d = 300 mm; A_{st} = 3 x (π/4) x 16^2 = 603.2 mm²; m = 19;

Neutral axis

Let the depth of N.A. be n. Equating the moment of the compressive area to the moment of *equivalent* area of steel, about N.A., we get
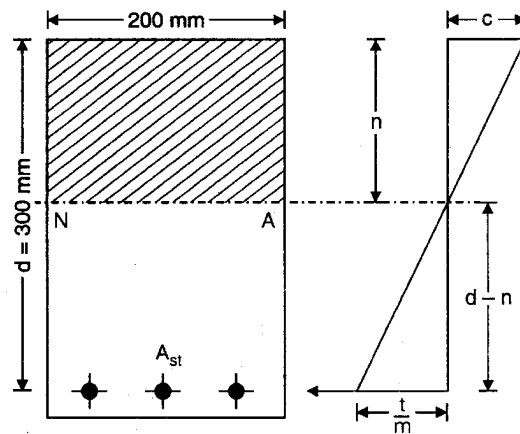
$$b.n.\frac{n}{2} = mA_{st}(d-n)$$

or $$100n^2 = 19(603.2)(300-n)$$

or $$n^2 + 114.6n - 34382 = 0$$

∴   n = 136.8 mm

Stress diagram

Maximum stress in concrete

Stress in steel = t = 120 N/mm^2 (Given)

∴ $$\frac{t}{m} = \frac{120}{19} = 6.316$$

From stress diagram

$$\frac{c}{n} = \frac{t/m}{d-n}$$

$$\frac{c}{136.8} = \frac{6.316}{300-136.8}$$

∴   c = 5.29 N/mm^2

Example

A rectangular, singly reinforced beam, 300 mm wide and 500 mm effective depth is used as a simply supported beam over an effective span of 6 m. The reinforcement consists of 4 bars of

20 mm diameter. If the beam carries a load of 12 kN/m, inclusive of the self-weight, determine the stresses developed in concrete and steel. Take m = 19.

Solution

Neutral axis and lever arm

Equating the moments of two areas about NA, we get

$$b \, x \, n \, x \frac{n}{2} = m A_{st}(d-n)$$

$$\therefore \qquad 300 \, x \frac{n^2}{2} = 19 \, x \, 1256.6 (500 - n)$$

$$\therefore \qquad n = 213.5 \text{ mm}$$

Lever arm $= a = d - \dfrac{n}{3} = 500 - \dfrac{213.5}{3} = 428.8 \, mm$

Maximum bending moment

It will be $\qquad \dfrac{wl^2}{8} = \dfrac{12(6)^2}{8} = 54 \, kN - m = 54 \, x \, 10^6 \, N - mm$

Moment of resistance

Let c be the compressive stress in concrete.

$$M_r = \frac{1}{2} c.n.b \, x \, a = \frac{1}{2} c \, x \, 300 \, x \, 213.5 \, x \, 428.8 = 13.732 \, x \, 10^6 \, c \, N - mm$$

Compressive stress in concrete

Equating moment of resistance to external bending moment,

$$13.732 \, x \, 10^6 \, c = 54 \, x \, 10^6$$

$$\therefore \qquad c = \frac{54}{13.732} = 3.93 \, N/mm^2$$

Stresses in steel

Method 1

If t is the corresponding stress in steel, we get

$$\frac{c}{n} = \frac{t/m}{d-n}$$

$$\therefore \qquad t = \frac{mc}{n}(d-n) = \frac{19 \, x \, 3.93}{213.5}(500 - 213.5) = 100.2 \, N/mm^2$$

Method 2

$$M_r = A_{st}.t.a = 1256.6 \, x \, t \, x \, 428.8$$

$\therefore$ $\quad$ $1256.6 \, x \, 428.8t = 54 \, x10^6$

$\therefore$ $\quad$ $t = 100.2 \text{ N/mm}^2$

Problem

A reinforced concrete beam has section 300 mm wide and 600 mm deep. The reinforcement consists of mild steel bars of 25 mm diameter with a cover of 50 mm to the centre of reinforcement. If it is subjected to a bending moment of 120 kN-m, determine the stresses developed in steel and concrete. Take m = 15.

Answer: 6.57 N/mm^2; 106 N/mm^2

Example

A reinforced concrete beam 200 mm x 400 mm effective depth is used over an effective span of 5 m. It is subjected to a uniformly distributed load of 5 kN/m inclusive of Its own weight. Find the necessary steel reinforcement at the centre of the span. Take allowable stresses in steel and concrete as 130 N/mm^2 and 4 N/mm^2 respectively, and m=16.

Solution

Given data

$c = \sigma_{cbc} = 4 \text{ N/mm}^2$; $t = \sigma_{st} = 130 \text{ N/mm}^2$; $m = 16$

Moment of resistance

$$M = \frac{wl^2}{8} = \frac{5x25}{8} = 15.625 \, kN - m = 15.625 x10^6 \, N - m$$

$$M_r = Rbd^2 = 15.625 \, x10^6$$

$\therefore$ $\quad$ $M_r = 0.488bd^2$ $\qquad\qquad\qquad\qquad$ (1)

For balanced section

$$k = \frac{mc}{mc+t} = \frac{16 \, x \, 4}{16 \, x \, 4 + 130} = 0.33$$

$$j = 1 - \frac{k}{3} = 1 - 0.11 = 0.89$$

$\therefore$ $\quad$ $R = \frac{1}{2} cjk = \frac{1}{2}(4)(0.89)(0.33) = 0.587$

$\therefore$ $\quad$ $M_r = 0.587bd^2$ $\qquad\qquad\qquad\qquad$ (2)

Neutral axis

The moment of resistance of the given beam has to be less than the moment of resistance of the critical section. Hence the steel reinforcement will be corresponding to an **under reinforced beam**. For such a section, the stress in steel will reach the maximum value of 130 N/mm^2.

Let   n = depth of N.A. of the actual section.

Then the corresponding stress in concrete is given by

$$\frac{c}{t/m} = \frac{n}{d-n}$$

∴ $$c = \frac{t}{m} \cdot \frac{n}{d-n} = \frac{130}{16} x \frac{n}{400-n}$$ (3)

Now $$M_r = \frac{1}{2} c.n.b \left(d - \frac{n}{3} \right)$$

∴ $$15.625 \, x 10^6 = \frac{1}{2} \left(\frac{130}{16} x \frac{n}{400-n} \right) n \, x \, 200 \left(400 - \frac{n}{3} \right)$$

∴ $$n^2 \frac{(1200-n)}{400-n} = 57700$$

This is a cubic equation. Solving by trial and error

         n = 121.8 mm

Stress in concrete

From (3)

$$c = \frac{130}{16} x \frac{121.8}{400-121.8} = 3.56 \, N/mm^2$$

Area of steel

         Total compression = total tension

∴ $$\frac{1}{2} cnb = A_{st}.t$$

∴ $$A_{st} = \frac{1}{2} \frac{cnb}{t} = \frac{1}{2} \cdot \frac{3.56 \, x 121.8 \, x 200}{130} = 333 \, mm^2$$

Problem

A reinforced concrete beam 200 mm x 400 mm effective depth is used over an effective span of 5 m. It is subjected to a uniformly distributed load of 7 kN/m inclusive of its own weight

Find the necessary steel reinforcement at the centre of the span. Take allowable stresses in steel and concrete as 130 N/mm² and 4 N/mm² respectively and m = 16.

Solution

Given data

$$c = \sigma_{cbc} = 4 \text{ N/mm}^2; \, t = \sigma_{st} = 130 \text{ N/mm}^2; \, m = 16$$

Moment of resistance

$$M = \frac{wL^2}{8} = \frac{7 x 25}{8} = 21.88 \, kN - m = 21.88 \, x 10^6 \, N - mm$$

∴
$$R = \frac{M_r}{bd^2} = \frac{M}{bd^2} = \frac{21.88 x 10^6}{200 \, x \, (400)^2} = 0.684$$

∴
$$M_r = 0.684bd^2 \tag{1}$$

Neutral axis

Since the moment of resistance of the given beam is greater than the moment of resistance of the critical section, the steel reinforcement will be corresponding to an over-reinforced beam. For such a section, the stress in concrete will reach the maximum value of 4 N/mm²

$$M_r = \frac{1}{2} cnb \left(d - \frac{n}{3} \right)$$

∴
$$21.88 x 10^6 = \frac{1}{2} x 4n \, x \, 200 \left(400 - \frac{n}{3} \right)$$

∴
$$n = 157.5 \text{ mm}$$

Stress in steel

$$\frac{t/m}{c} = \frac{d-n}{n}$$

∴
$$t = \left(\frac{d-n}{n} \right) mc = \frac{400 - 157.5}{157.5} x 16 x 4 = 98.5 \, N / mm^2$$

Area of steel

total tension = total compression

∴
$$t.A_{st} = \frac{1}{2} cn.b$$

∴
$$A_{st} = \frac{1}{2}.\frac{cnb}{t} = \frac{1}{2}.\frac{4 x 157.5 x 200}{98.5} = 639.6 \, mm^2$$

☞ *In this example of an over-reinforced section, the stress in steel is only 98.5 N/mm² which is very much less than the permissible value of 130 N/mm². Thus the strength of steel is not fully utilized. Such a design is undesirable and uneconomical. In circumstances where the dimensions of the beam are limited, and where the section has to develop greater moment of resistance than that of the balanced section, it is always desirable to design the section as doubly reinforced. In doubly reinforced section, steel reinforcement is placed in the compression zone also, which increases its moment of resistance.* **Hence a doubly reinforced section is always preferred over an over reinforced section.**

Example

Design a reinforced concrete beam subjected to a bending moment of 20 kN-m. Use M 20 concrete, and Fe 415 reinforcement. Keep the width of the beam equal to half the effective depth.

Solution

Choose data

For M 20 concrete, c = σ_{cbc} = 7 N/mm²; m = 13

For Fe 415 steel, σ_{st} = 230 N/mm².

Balanced section

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{13x7}{13x7+230} = 0.283$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.283}{3} = 0.906$$

$$R = \frac{1}{2}c.j.k = \frac{1}{2}(7)(0.906)(0.283) = 0.898$$

Moment of resistance

Now $\qquad M_r = Rbd^2 = R.\frac{d}{2}.d^2 = \frac{Rd^3}{2} = \frac{0.898}{2}d^3 = 0.449d^3 \, N - mm$

Depth of beam

Given BM = M = 20 kN-m = 20 x 10⁶ N-mm

∴ $\qquad$ M_r = M

$\qquad\qquad 0.449d^3 = 20 \, x10^6$

∴ $\qquad$ d = 354 mm

∴ $\qquad$ b = d/2 = 177 mm

Area of steel

$$A_{st} = \frac{M}{\sigma_{st} jd} = \frac{20 \times 10^6}{230 \times 0.906 \times 354} = 271.1 \, mm^2$$

Problem

Design a R.C. beam 10 carry a load of 6 kN/m inclusive of its own weight on an effective span of 6 m. Keep the breath to be 2/3rd of effective depth. The permissible stresses in concrete and steel are not to exceed 5 N/mm² and 140 N/mm² respectively. Take m = 18.

Answer: b = 240 mm; d = 360 mm; A_{st} = 616 mm²

Doubly Reinforced Beam

The moment of resistance of a balanced section is equal to Rbd². If a beam of specified dimensions (b x d) is required to resist a moment much greater than Rbd², there are two alternatives:

- to use an over-reinforced section, or
- to use doubly reinforced section

An over reinforced section is always uneconomical since the increase in the moment of resistance is not in proportion to the increase in the area of tensile reinforcement since the concrete, having reached maximum allowable stress, cannot take more additional load without adding compression steel.

The other alternative is to provide reinforcement in the compression side of the beam and thus to increase the moment of resistance of the beam beyond the value Rbd² for a singly reinforced balanced section. *Sections, reinforced with steel in compression and tension are known as doubly reinforced sections.*

A doubly reinforced section is generally provided under the following conditions;

- When the depth and breadth of the beam are restricted and it has to resist greater bending moment than a singly reinforced beam of that section would do.

- When the beam is continuous over several supports, the section of the beam at the supports is usually designed as a doubly reinforced section.

- When the member is subjected to eccentric loading.

- When the bending moment in the member reverses according to the loading conditions e.g., the wall of an underground R.C.C. storage reservoir, brackets etc.

- When the member is subjected to shocks, impact or accidental lateral thrust.

NEUTRAL AXIS

Following figure shows a doubly reinforced section.
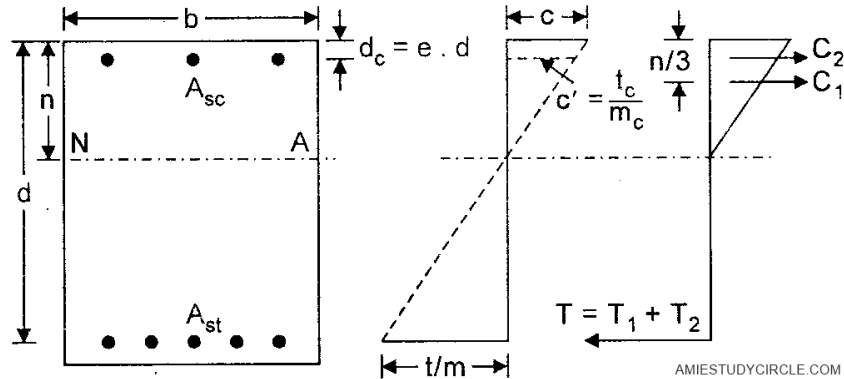
Let   b = breadth of the beam.

d = effective depth of the beam

d_c = depth of the centre of the compressive steel = e.d

e = compressive steel depth factor = d_c/d

c = maximum stress in concrete

t = maximum stress in steel

c' = stress in concrete surrounding compressive steel

t_c = stress in compressive steel = m_c .c' = 1.5 mc'

A_{st} = area of tensile steel

A_{sc} = area of compressive steel

m_c = modular ratio for compression zone = 1.5 m

If maximum stresses in concrete and tensile steel are known the N.A. can be located from stress diagram exactly in the same manner as in singly reinforced section:

$$\frac{c}{t/m} = \frac{n}{d-n} = \frac{kd}{d-kd} = \frac{k}{1-k}$$

∴ $$k = \frac{mc}{mc+t} = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

where σ_{cbc} and σ_{st} are the permissible stresses in concrete and steel respectively. Thus the presence of steel in compression zone *does not* affect the depth of N.A.

Also $$c' = c.\frac{(n-d_c)}{n}$$

Again neglecting the concrete in the tensile zone and equating the moment of the compressive area about N.A. to the moment of the tensile area about N.A. we get

$$\frac{bn^2}{2} + m_c A_{sc}(n-d_c) - A_{sc}(n-d_c) = mA_{st}(d-n)$$

MOMENT OF RESISTANCE

The moment of resistance of a doubly reinforced section can be determined by taking the moment of the compressive force about the centre of the tensile reinforcement.

Let, M = total bending moment to be resisted by the section = moment of resistance of the section = $M_1 + M_2$

M_1 = moment that can be developed by the balanced section, without compressive reinforcement = moment of the compressive force in concrete about the centre of tensile reinforcement.

M_2 = moment, in excess of M_1, that is to be provided by the compression steel = moment of the compressive force in steel, about the centre of the tensile reinforcement.

A_{st} = total tensile reinforcement = $A_{st1} + A_{st2}$

A_{st1} = tensile steel for the balanced section, corresponding to the moment M_1

A_{st2} = additional tensile steel necessary to develop the remaining moment M_2

Taking the moments of the compressive forces about the centre of the tensile steel, we get

$$M = bn \frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1)A_{sc}.c'(d - d_c)$$

$$M = b.kd.\frac{c}{2}\left(d - \frac{kd}{3}\right) + (m_c - 1)A_{sc}.c'(d - ed)$$

or

$$M = \frac{1}{2}jck.bd^2 + (m_c - 1)A_{sc}.c'd(1 - e)$$

or

$$M = \underbrace{Rbd^2}_{M_1} + \underbrace{c'd(m_c - 1)A_{sc}(1 - e)}_{M_2}$$

or $M = M_1 + M_2$

Now, $$A_{st1} = \frac{M_1}{t \, x \, lever \, arm} = \frac{M_1}{tjd} = \frac{M_1}{\sigma_{st}.jd}$$

and $$A_{st2} = \frac{M_2}{t \, x \, lever \, arm} = \frac{M_2}{t(d - d_c)}$$

where t may be equal to or less than σ_{st}.

The additional tensile area A_{st2} is actually required to balance the compressive steel. Hence A_{st2} can also alternatively be determined by equating the moment of the equivalent concrete area of the compressive steel to the moment of equivalent concrete area of the tensile steel A_{st2} about N.A.

Thus, $(m_c - 1)A_{sc}(n - d_c) = mA_{st2}(d - n)$

Example

AMIE(I)   STUDY CIRCLE(REGD.)

A FOCUSSED APPROACH

A beam section, 100 mm wide and 560 mm deep is reinforced with 4 bars of 25 mm diameter in the tensile zone and 4 bars of 12 mm diameter in the compression zone. The cover to the centre of both the reinforcement is 40 mm. Determine the moment of resistance of the section, if M 20 concrete and HYSD bars are used.

Solution

Given data

σ_{cbc} = 7 N/mm^2; σ_{st} = 230 N/mm^2; m = 13; m_c = 1.5m = 1.5 x 13 = 19.5
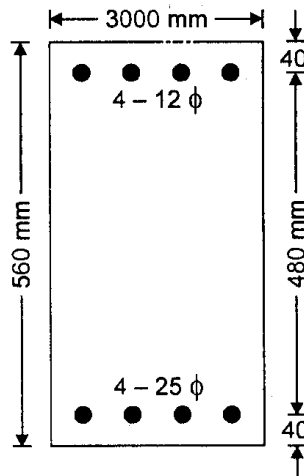
Area of compressive and tension steel

$$A_{sc} = 4.\frac{\pi}{4}(12)^2 = 452.4\,mm^2$$

$$A_{st} = 4.\frac{\pi}{4}(25)^2 = 1963.5\,mm^2$$

Neutral axis

Let us determine the position of the neutral axis by equating the moments of area of compressive concrete and the area of compressive steel expressed in equivalent concrete to the moment of the area of the concrete equivalent to the area of steel in tension, about the N.A.

$\therefore$ $\frac{300}{2}n^2 + (19.5-1)\,x\,452.4(n-40) = 13\,x\,1963.5(520-n)$

Solving   n = 208.7 mm

Critical neutral axis

Critical neutral axis is given by

$$n_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}.d = \frac{13 \times 7}{13 \times 7 + 230}(520) = 147.4\,mm$$

Stresses

Since the actual NA is below the critical NA, the stress in concrete will reach its maximum permissible value first.

$\therefore$ $c = \sigma_{cbc} = 7\ \text{N/mm}^2$

$$c' = c\frac{n - d_c}{n} = 7\,x\frac{208.7 - 40}{208.7} = 5.658\,N\,/\,mm^2$$

Moment of resistance

$$M_r = 300(208.7)\,x\,\frac{7}{2}\left(520 - \frac{208.7}{3}\right) + (19.5 - 1)(452.4)(5.658(520 - 40)$$

$$= 121.44 \times 10^6\ \text{N-mm} = 121.44\ \text{kN-m}$$

Problem

A beam 250 mm x 500 mm in section is reinforced with 3 bars of 14 mm diameter at top and 5 bars of 20 mm diameter at the bottom, each at an effective cover of 40 mm. Determine the moment of resistance of the beam section. Take the permissible stress in steel and concrete as 126 N/mm² and 5.2 N /mm² respectively, and m = 18.

Answer: 73.5 kN-m

Example

A doubly reinforced concrete beam is 400 mm wide and 600 mm deep to the centre of tensile reinforcement. The compression reinforcement consists of 4 bars of 16 mm diameter, and is placed with its centre at a depth of 40 mm from the top. The tensile reinforcement consists of 4 bars of 20 mm diameter. The section is subjected to a bending moment of 100 kN-m. Determine the stresses in concrete and steel. Take m = 16.
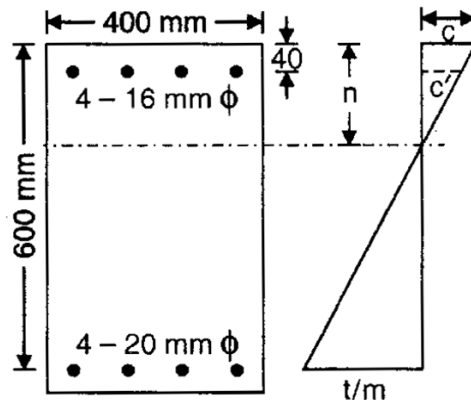
Solution

Area of compressive and tensile steels

$$A_{sc} = 4\left(\frac{\pi}{4}\right)(16)^2 = 805\,mm^2$$

$$A_{st} = 4\left(\frac{\pi}{4}\right)(20)^2 = 1250\,mm^2$$

Stress diagram

Neutral axis

Taking moment of equivalent areas about NA

$$\frac{bn^2}{2} + (m_c - 1)A_{sc}(n - d_c) = mA_{st}(d - n)$$

or

$$\frac{400}{2}n^2 + (1.5\,x16 - 1)(805)(n - 40) = 16(1250)(600 - n)$$

Solving   n = 173.8 mm

Stress in compression steel

Let the maximum stress in concrete be c. The stress c' in concrete surrounding the compression steel is given by

$$c' = \frac{n - d_c}{n}c = \frac{173.8 - 40}{173.8}c = 0.77c$$

Now

$$M_r = bn\frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1)A_{sc}.c'(d - d_c)$$

∴

$$100x10^6 = 400x173.8.\frac{c}{2}\left(600 - \frac{173.8}{3}\right) + (1.5x16 - 1)(805)x0.77c(600 - 40)$$

∴   c = 3.73 N/mm^2

Now   c' = 0.77c = 0.77 x 3.73 = 2.87 N/mm^2

∴ Stress in compression steel will be = 1.5mc' = 1.5 x 16 x 2.87 = 68.9 N/mm^2

Stress in tension steel

$$t = \frac{cm(d - n)}{n} = \frac{3.73\,x16(600 - 173.8)}{173.8} = 146.3\,N/mm^2$$

Problem

A doubly reinforced concrete beam is 250 mm wide and 500 mm deep from the compression edge to the centre of tensile steel. The area of compression and tensile steel are both 1300 mm² each. The centre of compression steel is 50 mm from the compression edge. If the beam is subjected to a total bending moment of 70 kN-m, determine the stress in concrete, and tension and compression steel. Take m = 18.

*Answer: $c = 3.56 \, N/mm^2$; $t = 123.8 \, N/mm^2$; $t_c = 68.3 \, N/mm^2$*

Example

A doubly reinforced rectangular beam is 240 mm wide and 500 mm deep. If the limiting stresses in concrete and steel are 5 N/mm² and 230 N/mm² respectively determine the steel areas for bending moment of 80 kN-m. Assume that steel is buried on both faces with its centre 40 mm from either face. Take m = 19.

Solution

Given data

$c = \sigma_{cbc} = 5 \, N/mm^2$; $t = \sigma_{st} = 230 \, N/mm^2$
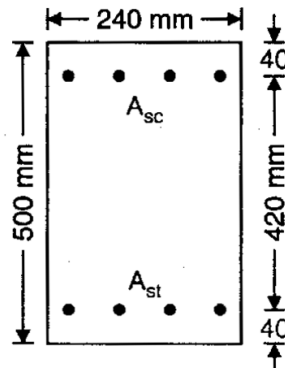
Neutral axis

$$n = n_c = \frac{mc}{mc+t}.d = \frac{19x5}{19x5+230}x460 = 134.5 \, mm$$

Area of compression steel

Max stress in concrete = c = 5 N/mm²

$$c' = \frac{n-d_c}{n}.c = \frac{134.5-40}{134.5}x5 = 3.513 \, N/mm^2$$

Now $M_r = b.n.\frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c-1)A_{sc}.c'(d-d_c)$

or $\quad 80x10^6 = 240x134.5x\dfrac{5}{2}\left(460 - \dfrac{134.5}{3}\right) + (1.5x19-1)A_{sc}x3.513(460-40)$

$\therefore \quad A_{sc} = 1146 \text{ mm}^2$

Area of tension steel

Total compression = total tension

$\therefore \quad bn\dfrac{c}{2} + (m_c - 1)A_{sc}.c' = A_{st}.t$

or $\quad 240x134.5x\dfrac{5}{2} + (1.5x19-1)(1146)(3.513) = A_{st}(230)$

$\therefore \quad A_{st} = 832 \text{ mm}^2$

☞ *It is found here that $A_{sc} > A_{st}$. Such a design is uneconomical because the HYSD bars (A_{sc}) provided in compression zone are highly under-stressed. For such a circumstance, it is better to re-design the beam.*

Problem

A doubly reinforced rectangular beam is 300 mm wide and 450 mm deep and is subjected to a bending moment of 90 kN-m. If the limiting stresses in concrete and steel are 5 N/mm^2 and N/mm^2. determine the steel areas. Assume that steel is buried on both faces, with its centre mm from either face. Take m = 18.

Answer: $A_{st} = 1770 \text{ mm}^2$; $A_{sc} = 1300 \text{ mm}^2$

ASSIGNMENT

Q.1. (AMIE W13, S15, 10 marks): A RCC beam is 30 cm wide and 70 cm deep. The centres of the steel are 5 cm from the respective edges. Determine the area of steel in tension and compression zone for a bending moment of 13×10^5 kg-cm. The beam is doubly reinforced. Assume the limiting stress in concrete and steel are 50 kg/cm^2 and 1400 kg/cm^2, respectively. Given modular ratio = m = 18.

Answer: A_{sc} = 540.87 mm^2; A_{st} = 1626.61 mm^2

Q.2. (AMIE W15, 5 marks): Briefly describe the major steps for design of axially loaded tension member.

Q.3. (AMIE W15, 14 marks): A simply-supported RCC beam, 300 mm wide and 600 mm deep, has four bars of 20 mm diameter as tension reinforcement, the centre of the bars being 50 mm from the bottom of the beam. Determine the uniformly distributed load the beam can carry over an effective span of 6 m. Take the permissible stresses in concrete and steel as 5 N/mm^2 and 140 N/mm^2, respectively. Given that m = 18.

Answer: 14.63 kN/m

Q.4. (AMIE S16, 6 marks): Define effective cover, development length of rebar and balanced beam section.

Q.5. (AMIE S16, 14 marks): A simply supported RCC beam is 400 mm and 800 mm. Assume effective cover of 50 mm and calculate the minimum reinforcement required and that required for a balanced section. Also, write the procedure to calculate shear force requirement. Use M20 grade concrete and Fe 415 steel.

Answer: $A_{st,bal}$ = 1320 mm^2;

Q.6. (AMIE S18, 20 marks): Design a rectangular beam 300 mm in width and 500 mm in depth to carry a load of 25 kN/m for a span of 5 m. Check if compression reinforcement is required. M20 concrete and Fe 415 steel bars are to be used.

Q.7. (AMIE W18, 20 marks): A singly reinforced concrete beam with an effective span of 4 m has a rectangular cross section of 250 mm x 550 mm. It is reinforced with 3 bars of 10 mm dia Fe 415 bars at an effective depth of 500 mm. The self weight of the beam together with dead load is 4 kN/m. Calculate the maximum permissible live load on the beam. Assume M20 grade concrete.

Answer: 5041.54 N/m^2

Important Note

Candidates intend to appear in Section B examination, are permitted to use reference material for different engineering discipline as indicated below in the Examination Hall. In case the Examination Centre is not in a position to provide the required reference materials on request, candidates may use their own copy, subject to prior verification by the Officer in-Charge/ Invigilator to the extent that no extraneous matter is written inside pages of the concerned reference material:

- IS 800-1984
- IS 456-2000
- IS 1893
- IS 13920
- IS 4326
- IS 13827

- IS 13935
- SP 6(1) and SP (16)
- IRC - 6- 1966 Section III, Loads and Stresses Steel Tables

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